Antiderivatives Formula Sheet

1 Basic Antiderivatives

- If $f(x) = a$ then $F(x) = ax + C$
- If $f(x) = x^a$ then $F(x) = \frac{x^{a+1}}{a+1} + C$ (unless $a = -1$)
- If $f(x) = \frac{1}{x}$ then $F(x) = \ln(x) + C$
- If $f(x) = e^x$ then $F(x) = e^x + C$
- If $f(x) = \cos(x)$ then $F(x) = \sin(x) + C$
- If $f(x) = \sin(x)$ then $F(x) = -\cos(x) + C$
- If $f(x) = \sec^2(x)$ then $F(x) = \tan(x) + C$

2 Antiderivatives Rules

If the antiderivative of $f(x)$ is $F(x)$, and the antiderivative of $g(x)$ is $G(x)$ then

- The antiderivative of $af(x) + bg(x)$ is $aF(x) + bG(x)$ (for any a, b)
- The antiderivative of $f(ax + b)$ is $\frac{1}{a}F(ax + b)$

As a result:

- If $f(x) = (dx + b)^a$ then $F(x) = \frac{1}{d}\frac{(dx+b)^{a+1}}{a+1} + C$ (unless $a = -1$)
- If $f(x) = \frac{1}{ax+b}$ then $F(x) = \frac{1}{a}\ln(ax + b) + C$
- If $f(x) = e^{ax+b}$ then $F(x) = \frac{1}{a}e^{ax+b} + C$
- If $f(x) = \cos(ax + b)$ then $F(x) = \frac{1}{a}\sin(ax + b) + C$
- If $f(x) = \sin(ax + b)$ then $F(x) = -\frac{1}{a}\cos(ax + b) + C$
- If $f(x) = \sec^2(ax + b)$ then $F(x) = \frac{1}{a}\tan(ax + b) + C$

3 Intermediate Rules

- The antiderivative of $\frac{f'(x)}{f(x)}$ is $\ln[f(x)] + C$
- The antiderivative of $f'(x)f^a(x)$ is $\frac{f^{a+1}(x)}{a+1}$. The following are useful "special cases" of this rule:
  - The antiderivative of $f'(x)f(x)$ is $\frac{1}{2}f^2(x)$
  - The antiderivative of $\frac{f'(x)}{\sqrt{f(x)}}$ is $2\sqrt{f(x)}$
  - The antiderivative of $\frac{f'(x)}{f^2(x)}$ is $-\frac{1}{f(x)}$
- The antiderivative of $f'(x)e^{f(x)}$ is $e^{f(x)} + C$
- The antiderivative of $f'(x)\cos[f(x)]$ is $\sin[f(x)] + C$
- The antiderivative of $f'(x)\sin[f(x)]$ is $-\cos[f(x)] + C$