Logarithm Formulas

Expansion/Contraction Properties of Logarithms

These rules are used to write a single complicated logarithm as several simpler logarithms (called "expanding") or several simple logarithms as a single complicated logarithm (called "contracting"). Notice that these rules work for any base.

$\log_a(xy) = \log_a(x) + \log_a(y)$ (multiplication inside can be turned into addition outside, and vice versa.)

$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$ (division inside can be turned into subtraction outside, and vice versa)

$\log_a(x^n) = n \cdot \log_a(x)$ (an exponent on everything inside can be moved out front, and vice versa)

Change of Base Formula

This formula is used to change a less helpful base to a more helpful one (generally base 10 or base e, since these appear on your calculator, but you can change to any base). In the formula below, a is the current base of your logarithm, and b is the base you would like to have instead.

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

Cancellation Properties of Logarithms

These rules are used to solve for x when x is an exponent or is trapped inside a logarithm. Notice that these rules work for any base.

$\log_a(a^x) = x$ (this allows you to solve for x whenever it is in the exponent)

$a^{\log_a(x)} = x$ (this allows you to solve for x whenever it is inside a logarithm)

Logarithm Problems

1. Expand each expression (use expansion properties to expand as much as possible).

a. $\log_3(xy)$
Solution:

$$\log_3(xy) = \log_3(x) + \log_3(y) \qquad \text{(multiplication rule)}$$

b. $\log_7\left(\frac{x}{y}\right)$
Solution:

$$\log_7\left(\frac{x}{y}\right) = \log_7(x) - \log_7(y) \qquad \text{(division rule)}$$

c. $\ln(x^5)$
Solution:

$$\ln(x^5) = 5\ln(x) \qquad \text{(exponent rule)}$$